LESSONS IN FORM.

TO PREPARE FOR
AND TO ACCOMPANY

THE STUDY OF NUMBER.

FORM LESSONS.

TO PREPARE FOR AND TO ACCOMPANY

THE STUDY OF NUMBER.

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PREFACE.

In the first grade, this book is designed to aid teachers in systematizing their oral lessons. In the other grades, the book should be in the hands of each pupil. The constant test of the pupil's observation must be the accuracy of his expression. The questions asked are for the pupil to answer, and not the teacher. When his descriptions are not correct, he, through renewed observation, must discover and correct his error. It is not expected that all that can be discovered in the various forms will be seen by first or second-year pupils, but many tests have shown that the work is well adapted for these grades, while it furnishes sufficient material to profitably engage the attention of pupils of any grade. As an aid to mathematical investigation and for securing clear and concise expression, it will, I think, be found helpful even in the high school before beginning the study of scientific geometry, if pupils have not had previous training of this character.

Where the programme is so full that there is not time to give special attention to teaching form, the lessons can be used as a means of teaching language. As the forms to be compared are definite, and demand accuracy of expression in description, and as all inaccuracy of statement can be readily detected, no study furnishes a better basis for language lessons. In the written exercises, penmanship, spelling, and punctuation can be taught.

The attention of the reader is respectfully invited to a consideration of the remarks on page 73, on the value of form lessons as a preparation for number study, and to the opinions of some eminent educators and thinkers on this subject.

ENGLEWOOD, ILL. W. W. SPEER.

OBSERVATION LESSONS.

"Lessons especially designed to cultivate the power and habit of observation appear to be less widely used than is desirable. It is probable that the erroneous methods of teaching, too often employed in such lessons, have led to meager results and consequent distrust of this branch of primary school work. The skill required to teach such lessons properly is apparently less common than skill in teaching other branches. The mistake often made is that of supposing a pupil is learning how to *observe* when he is merely listening to what his teacher *tells* him to *remember* about an object he may be looking at. So-called object lessons taught by such false methods have no tendency to cultivate the power and habit of observation, but rather to confuse and stultify the child's mind. On the other hand, observation lessons, *in which the children really do the observing*, not only develop the observing powers, but also furnish the children's minds with a stock of clear ideas which constitute the best possible material for language work."

—*Report of Massachusetts Teachers' Association, 1887.*

LESSONS IN FORM.

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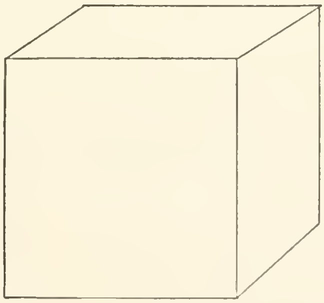
THE STUDY OF NUMBER.

PART FIRST.

The first exercises are to cultivate the habit of observation, to teach direction and position, and to train pupils to associate the terms to be used in the comparisons which follow with the corresponding ideas.

SURFACES, LINES AND POINTS.



Give each pupil a cubic inch, or have a large cube placed so that the class can observe it.

Directions and Questions for Pupils:

Place your finger on the surface of the cube.
Place your hand on the surface of your desk.

Of what object do you see the surface?

Recall objects that you have seen at home that have surfaces.

Find other surfaces in the room.

Of what object at home are you thinking that has a surface?

Place your finger on one of the edges, or lines, of the cube. On one of the edges, or lines, of your desk.

Find other lines, or edges, in the room.

Of what have you found an edge, or a line?

Name five objects that you have seen outside of the school-room that have edges or lines.

Name five objects that you have seen outside the school-room whose surface is not bounded by lines. *Example:* The surface of a croquet ball is not bounded by lines.

Drawing.—Look at the cube and draw four lines each as long as an edge of the cube.

Use an edge of the cube and measure the lines you have drawn.

Draw again and measure.

Observe a face of the cube and draw it. Continue drawing and measuring until you can represent a face quite accurately.

Place a finger on one of the corners, or points, of the cube. On a corner, or point, of your desk.

Find other points in the room.

Of what have you found a point?

Place a finger on one of the lines of the cube.

What is at the end of this line?

Can you place a finger on a point which is at the end of one of the lines of the blackboard?

OPPOSITE, ADJACENT AND PARALLEL LINES.*

The upper and lower lines, or edges, of the blackboard are opposite lines of the blackboard. Can you find other edges, or lines, of the blackboard that are opposite?

Point to the other lines of the blackboard that are opposite.

Place your finger on two lines of the cube that are opposite.

Find opposite lines of your book.

Find other opposite lines in the room.

Of what have you found opposite lines?

Drawing.—Look at two of the opposite lines of the cube and draw two lines as long and as far apart.

Measure and see if they are the same length and as far apart as the opposite lines in the face of the cube. Draw again and measure.

In what direction do the opposite lines of the cube extend?

Find other lines in the room that extend in the same direction.

Of what have you found lines extending in the same direction?

Do the lines you drew extend in the same direction?

Find two lines of the blackboard that meet.

Touch two edges, or lines, of the cube that meet.

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*If words are taught in connection with the ideas, primary pupils will have no difficulty, because of their length, in learning to use parallel, parallelogram, perimeter, rectangle, rectangular, blackboard, grandfather, or parallelopiped. It is not wise to teach one set of terms in the primary and another in the higher grades. Teach the right terms from the beginning, so that pupils may learn to think in the proper language.

Lines that meet are adjacent lines. Find other adjacent lines in the cube.

Find other adjacent lines in the room.

Of what have you found adjacent lines, or edges?

Drawing.— Draw a pair of adjacent lines, making each line an inch long. Draw another pair, making each line two inches long. Draw another pair, making each line three inches long.

Measure each pair and draw again.

Find opposite lines and adjacent lines in the room and tell whether they are opposite or adjacent. *Example:* This line and that line of the door are opposite.

Lines that extend in the same direction are parallel. Find parallel lines of the cube. Of a book. Of a chalk box.

Drawing.— Draw three pairs of parallel lines each an inch long. Measure and draw again.

Do lines need to be of the same length that they may extend in the same direction?

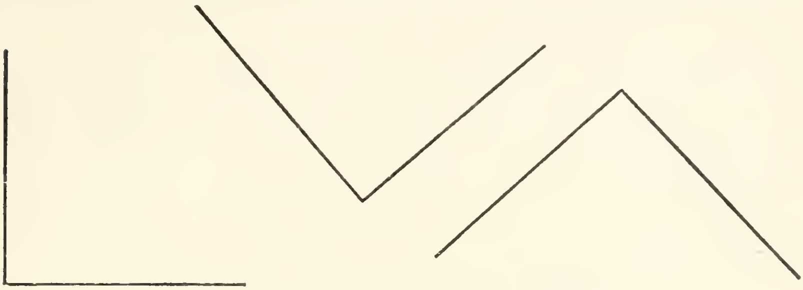
Do they need to be of the same length that they may be parallel?

When are lines parallel?

Can you find opposite lines in the room that are not parallel?

Suggestion: If pupils can not find opposite lines in the room that are not parallel, draw several quadrilaterals on the blackboard of which one or both pairs of opposite lines do not extend in the same direction. Have pupils point to lines of the figures that are opposite but not parallel.

ANGLES AND PERPENDICULAR LINES.



THESE ARE PERPENDICULAR LINES.

Find lines in the room that are perpendicular to each other.

How many pairs of perpendicular lines are there in the blackboard?

ANS. The edges of the blackboard form four pairs of perpendicular lines.

How many pairs of perpendicular lines are there in one face of the cube?

How many pairs of opposite lines are there in one face of the cube?

How many pairs of opposite lines are there in one of the walls of the room?

REMARK: When the attention of pupils is first called to the fact that lines may extend in different directions, or form angles, you will do them a service by using for the first illustration lines that do not intersect. A majority of pupils, who think they know what an angle is, are very sure that unless the lines do intersect there is no angle. This prejudice would not exist had their first impression of an angle been gained when considering the difference in direction of lines that did not intersect.

Point to lines extending in the same direction.

Point to lines that extend in different directions.

When two lines extend in different directions, *the difference in direction* is an angle.

Find five angles in the room. When you point to an angle, say that the difference in direction of this line and that line (indicating the direction each line extends, by moving pointer or hand) is an angle.

Hold two sticks or slats so that they are perpendicular to each other.

The difference in direction of two perpendicular lines is a right angle.

What is a right angle?

Find perpendicular lines, and say that the difference in direction of these perpendicular lines is a right angle.

Hold two splints so that the difference in direction which they indicate is not so great as a right angle.

When the difference in direction of two lines is not so great as that of a right angle, the difference in direction is called an acute angle.

Find in the room differences in direction less than a right angle. What is a difference in direction less than a right angle called?

Hold the slats or splints so that the difference in direction is greater than a right angle.

When the difference in direction of two lines is greater than the difference in direction of two perpendicular lines it is called an obtuse angle.

What is an obtuse angle?

Ans. An obtuse angle is a difference in direction greater than a right angle.

Find obtuse angles in the room and tell why they are obtuse. *Example:* The difference in direction of this line and that line is an obtuse angle because it is greater than a right angle.

How many differences in direction can you represent with two lines? With three lines? With four lines?

Represent three right angles by drawing lines.

Can you draw two right angles so that the difference in direction of one is greater than the difference in direction of the other?

If you make long lines in representing a right angle, is the difference in direction greater than when represented by short lines?

Why not?

Can you make two right angles with two lines?

Can you make three right angles with two lines?

Can you make four with two lines?

How many right angles can you make with three lines?

Draw *three* acute angles.

Can you draw two acute angles so that the difference in direction in one shall be greater than the difference in direction of the other?

Draw three acute angles of different sizes.

Draw three obtuse angles.

Can you draw obtuse angles of different sizes, or magnitudes?

If you increase the length of the lines of an obtuse angle does it increase the difference in direction? Does it increase the size of the angle? Why not?

There are what three kinds of angles?

What is a right angle? An acute angle? An obtuse angle? *

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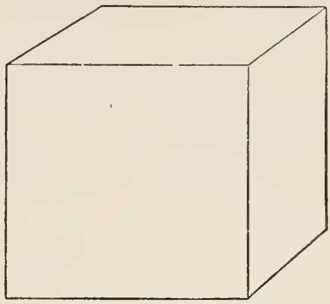
* Remarks: Two straight lines may have either of two relative positions.

1st. They may extend in the same direction.
2d. They may extend in different directions.

DIRECTION AND POSITION.



Place your finger on the front face of the cube, that is, the face next to you. On the back face. On

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In the first case, instead of saying that the lines extend in the same direction, it is customary to say the lines are parallel. One form of expressing this relation means no more nor less than the other. Parts of the same line thought of as distinct lines are parallel, for they can be thought of as extending in the same direction.

In defining parallel lines, do not add either of the following statements to the definition, namely: If produced, they would never meet; or, they are everywhere equally distant. It is not true that, when the parallels are parts of the same straight line, they would never meet if produced, and, it is an inference, from the definition proper, that they would never meet if they are not parts of the same line. An inference should not be a part of a definition. That parallels are everywhere equally distant, if they are not parts of the same straight line, is a proposition to be demonstrated in geometry, and it ought not to be made a part of a definition.

The second relation of lines, that of extending in different directions, is called an angle. An angle is simply a difference in direction. It may be a difference in direction of two lines, two or more surfaces, of two persons walking, etc. It is not necessary that the lines intersect in order that they indicate a difference in direction. Nothing is added to the difference in direction by the intersection. If the lines forming the angle do not intersect, and are in the same plane, it is an inference that they will intersect if produced in the direction of their convergence, and another inference that, if produced in the direction of their divergence, they will not intersect.

In order to measure an angle, or to prove it equal to another angle by superposition, the lines indicating the difference in direction must intersect, and, therefore, lie in the same plane.

The importance of fixing right ideas of the two relations of lines will be recognized when it is understood that all of the reasoning in both plane and solid geometry is based on a comparison of the length of lines, the sameness of direction or difference in direction of lines. Hazy impressions of these elementary ideas will make the reasoning based on such impressions lack in clearness.

the left-hand face. On the upper face, or base. On the right-hand face. On the upper right-hand corner in front. On the upper left-hand corner at the back. On the lower left hand corner in front. On the upper edge of the front face. On the upper edge of the left-hand face.

SUGGESTIONS: Have one pupil give a direction for pointing or placing a finger, and other pupils follow directions. Have other pupils give similar directions.

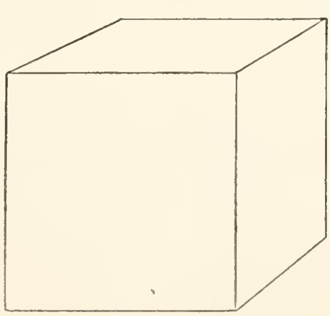
Have pupils point to different corners of objects and tell where each is. *Example:* That is the upper right hand corner in the front part of the room.

Place a finger on different points and faces of the cube or other objects, and have pupils tell where it is.

Have pupils point to different corners of the room and tell to what corner they are pointing. To different windows in the room and tell where each is. *Example:* That is the east window in the north wall. To different pupils and tell where each is. *Example:* Mary Warner sits on the third seat in the second row from the right.

Review exercises until pupils can follow directions in placing finger on face, line, or point designated, and can give directions clearly and without hesitation.

THE SURFACES, LINES AND POINTS OF A CUBE.



Count the surfaces of a cubic inch.

How many surfaces has a cubic inch?

How many surfaces has a chalk box?

Find other objects in the room that have six surfaces.

What object have you found that has six surfaces?

What objects have you seen at home or in going to and from the school that have six surfaces?

How many surfaces has the school-room?

(Count the walls, ceiling and floor.)

Place a finger on the upper face, or base, of the cubic inch.

Count the edges of the upper base.

How many edges has the upper base?

How many edges has the lower base?

Point toward the zenith, or the point in the heavens directly over your head.

Point toward the center of the earth.

An upright line, or a line which extends from the zenith toward the center of the earth, is a vertical line.

From what and toward what does a vertical line extend?

Find vertical lines in the school-room. *Example:* One of the right-hand edges of the door is an up-and-down line, or a vertical line.

Recall objects that are in a vertical position. *Example:* Some telegraph poles are in a vertical position.

Can you find edges of the cube that are vertical?

How many lines of the cube are vertical?

Point toward the horizon.

Find lines in the room that extend toward the

horizon. *Example:* The upper edge of the blackboard extends toward the horizon.

Lines that extend toward the horizon are horizontal lines. Find horizontal lines in the room. What lines of a window are horizontal?

In what direction do horizontal lines extend?

If you place a cube so that four of its edges are vertical, how many horizontal edges will it have?

Can you hold a cube so that none of its edges will be parallel? Why not? Can you hold a cube so that none of its edges will be perpendicular to each other?

Does it change the relation of the lines of a cube to each other to hold the cube in different positions?

Can you hold a cube so that its edges will be neither vertical nor horizontal?

How many lines has the cubic inch?

How many lines has a cubic foot?

Name other objects that have twelve lines, or edges.

How many lines bound the ceiling of the room?

How many lines bound the floor of the room?

How many vertical edges have the walls of the room?

How many corners, or points, has the cubic inch?

How many points has the upper base?

How many points has the lower base?

How many points has a cubic foot?

How many points has a brick?

How many lines meet in one of the points of the cubic inch?

In how many directions do the three edges extend from one point?

How many pairs of parallel lines are there in one face of the cube?

How many surfaces has the cube?

How many two pairs of parallel lines has the cube?

How many square inches are there in the surface of the cubic inch?

Can you tell eight things that are true of the cubic inch?

What is the shape of one of the surfaces of the cubic inch?

Find other squares in the room.

What object have you found that has a square surface?

Can you recall any squares that you have seen at home? *Examples:* The tops of some collar boxes are square. The bottoms of some salt cellars are square.

Can you recall any squares that you have seen in going to and from school?

Review, beginning with surfaces, lines and points of a cube.

Drawing.—Draw a two-inch square. Use a ruler and measure. Erase. Draw and measure again. Erase. Use the ruler and draw a two-inch square.*

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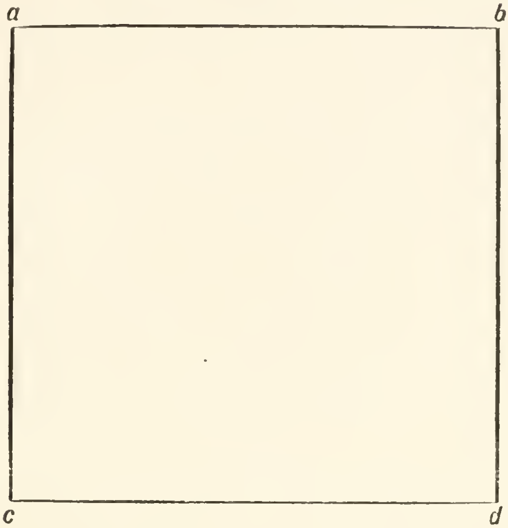
*Remark: Many of the pupils, in their efforts to make two-inch squares or to represent anything, will not do well at first if the drawing and not the effort is considered. The effort made to represent the thing observed or recalled is worth a thousand times more than the drawing, which is the result of the effort. The question is not, Are they making accurate and beautiful drawings? but, are they forming habits of observation?

Drawing furnishes a means of expressing ideas, and man first resorted to it for that purpose; but when it is perverted and fails to accomplish this purpose, it does not produce the best results. Any method that teaches

THE SQUARE.



Tell as many things as you can about the two-inch square.

QUESTIONS ON THE TWO-INCH SQUARE.

1. How many lines bound the square?
2. How many points has the square?
3. How many pairs of parallel lines has the square? How do parallel lines extend?

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words before ideas is radically wrong, and any method that teaches drawing without using it as a means of expressing or representing ideas, is radically wrong, because it leaves out that which stimulates and develops the powers of the mind. Reproducing a line without considering its length or direction does very little to increase one's power. That training which leads pupils to be imitators only does little to develop thought and action. Drawing ought to teach seeing, doing, and knowing. Drawing ought to cultivate the hand and the eye, and increase the knowledge of the object represented.

"As the first step in drawing is to learn to see correctly, it is evident that all the exercises, both in gifts and occupations, prepare for the use of the pencil and chalk. As the mediation of word and object drawing is of vast importance in its reaction on the mind, and as the soul of all technical processes, it is the indispensable basis of industrial education."

SUSAN E. BLOW.

4. How many pairs of adjacent lines has the square? (Each line is counted twice.)

5. How many pairs of perpendicular lines are there in the square?

6. The difference in direction of two perpendicular lines is what kind of an angle?

7. How many right angles are there in the square?

8. How many inches in the boundary, or perimeter, of the two-inch square?

"That which the pupil knows thoroughly contains an explanation of what he does not know."

DIRECTION AND POSITION.

To Teacher: Give each pupil a 4 inch square, and have him place the letter *a* near the upper left-hand point at the back, *b* near the right-hand point at the back, *c* near the left-hand point in front, and *d* near the right-hand point in front.

QUESTIONS.

1. In what part of the square is the point *a*? Ans. The point *a* is in the left-hand point at the back.

2. Where is the point *c*? The point *b*? The point *d*?

3. Where is line *a b* of the square? Ans. The line *a b* is the line at the back of the square.

4. Where is the line *c d*? The line *a c*? The line *b d*?

5. The points *c* and *b* are opposite points of the square. What other points of the square are opposite?

6. The line *a b* extends in the same direction as what line?

7. What are lines called that extend in the same direction?

SUGGESTION: Draw a square foot on the blackboard. Write the letter *a* near the upper left-hand point, *b* near the upper right-hand point, *c* near the lower left-hand point, and *d* near the lower right-hand point.

1. Which is the point *a*? ANS. It is the upper left-hand point of the sq. ft.

2. Which is the point *d*? The point *b*? The point *c*?

3. Which is the line *a b* of the sq. ft.?

4. Which is the line *a c* of the sq. ft.?

5. Close your eyes. Near what point is each letter in the sq. ft.? The letter *d* is near the lower right-hand point of the sq. ft.

SUGGESTION: Have pupils tell as many different things as they can about the positions of lines and points of the blackboard, door, etc., without questioning them.

6. Call the edge of your desk next to you the front edge.

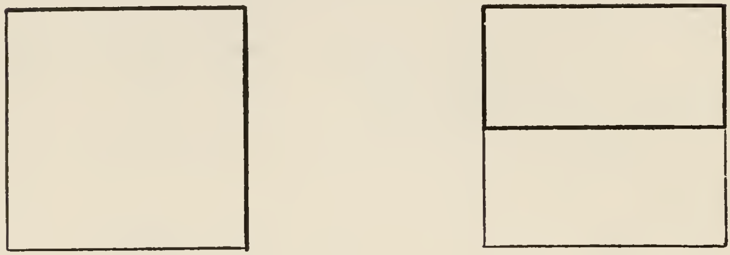
7. Place your hand on the front edge of your desk. On th edge at the back. On the middle of the right edge.

8. Toucn the left corner in front. The right corner at the back. Place your hand on the middle of the edge at the back.

SUGGESTION TO TEACHER: Have pupils give directions for other pupils to follow. Giving directions will force pupils to express themselves with precision. The necessity of saying exactly what they mean will make the exercise a valuable language lesson.

REVIEW FROM THE BEGINNING OF THE BOOK.

EXERCISES IN COMPARISON.*



COMPARISON OF THE SQUARE RECTANGLE WITH THE OBLONG RECTANGLE.

REMARK: The heavy lines, in the different cuts, indicate the forms to be compared made by folding the square.

Get kindergarten 4-inch squares, or cut 4-inch squares out of paper. Give each pupil two of these squares.

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* "It is by comparisons that we ascertain the difference which exists between things, and it is by comparisons, also, that we ascertain the general features of things, and it is by comparisons that we reach general propositions. In fact, comparisons are at the bottom of all philosophy. Without comparisons we never could go beyond the knowledge of isolated, disconnected facts. Now, do you not see what importance there must be in such training,—how it will awaken the faculties, how it will develop them, how it will be suggestive of further inquiries and further comparisons; and as soon as one has begun that sort of study there is no longer any dullness in it. Once imbued with the delight of studying the objects of nature, the student only feels that his time is too limited in proportion to his desire for more knowledge. And I say that we can in this way become better aquainted with ourselves. . . .

"The difficult art of thinking can be acquired by this method in a more rapid way than any other. When we study logic or mental philosophy in text-books, which we commit to memory, it is not the mind which we cultivate; it is the memory alone. The mind may come in, but if it does in that method it is only in an accessory way. But if we learn to think, by unfolding thoughts ourselves from the examination of objects brought before us, then we acquire them for ourselves, and we acquire the ability of applying our thoughts in life." AGASSIZ.

Direction for folding the square into the oblong rectangle:

Place one of the squares so that its front edge will extend in the same direction as the front edge of the desk. Fold the paper so that the front edge will lie along or coincide with the edge at the back. Crease the paper. Place the oblong rectangle formed near the square for comparison.

REMARK: When the pupils are folding papers they should be required to keep them in the same relative position. If they do not they can not follow directions. They should not lift the papers off the desks when folding.

1. Find the likenesses.
2. Find the differences.
3. Find square rectangles in room.
4. Recall square rectangles that you have seen at home or in going to and from school.
5. Find oblong rectangles in the room. *Example:* One of the windows is the shape of an oblong rectangle.
6. Recall oblong rectangles that you have seen. *Example:* One of the surfaces of a brick is an oblong rectangle.
7. In going to and from school or at home, you may find five square rectangles, and five oblong rectangles, and in to-morrow's recitation you may tell me the names of the objects that have square surfaces and those that have oblong rectangular surfaces.

The following questions are to be answered by pupils after they have found all the likenesses and differences they can in comparing the forms above:

REMARK: Pupils should be encouraged to give the names of the colors of the different papers used in the comparisons and to

find like colors in the room, and to recall objects that are similar in color.

QUESTIONS—LIKENESSES.

1. Each of the forms is what?

ANS. Each of the forms is a surface, or they are each surfaces.

2. Each surface has how many lines?
3. Each has how many points?
4. Each has how many pairs of parallel lines?
5. Each has how many pairs of adjacent lines?
6. Each has how many pairs of perpendicular lines?
7. Each has how many right angles?
8. Each has how many pairs of opposite points?
9. One of the long lines of the oblong rectangle is equal to what in the square?
10. The sum of the two short lines of the oblong rectangle is equal to what?
11. The opposite lines in each are what?
12. Give the likenesses without being questioned.

REMARKS: In elementary work, it is not well to spend time arguing with pupils, or trying to force your views upon them, even if you are in the right. You can not correct false notions or narrow views unless there are ideas enough in the pupil's mind to enable him to comprehend what you say. If you develop the discriminating power of the pupil by training him to observe, he will, in time, see for himself what you wish him to see. In observing the square, one pupil may see only two pair of perpendicular lines, while another sees the four. The former thinks that, as there are only four lines in the square, there can not be more than two pair of perpendicular lines. Trying to convince the first pupil that he is wrong may be worse than a waste of time. To-day, he sees two pair; leave his mind free, and to-morrow he may discover the four.

DIFFERENCES.

1. The square rectangle is bounded by how many lines? The oblong rectangle is bounded by two equal long lines, and by what?

2. One of the short lines of the oblong rectangle is equal to what part of one of the lines of the square rectangle?

3. The oblong rectangle is equal to what part of the square rectangle?

4. The sum of the lines of the oblong rectangle is equal to how many of the lines of the square?

5. How many inches are there in the boundary of each form?

6. Can you find rectangles in the room that are not one-half of a square rectangle?

7. Are the short sides of oblong rectangles always equal to one-half their long sides?

8. What is the direction for folding the square into the oblong rectangle?

Cutting.—Direction for pupils:

Cut out of paper an inch, a two-inch, a three-inch, and a four-inch square. Bring them to the class to-morrow. You may cut and measure as many as you wish before cutting those you bring to the class. Pin the squares you bring to the class together and write your name on one of them.*

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* "Through their own productions children are slowly awakening to facts of form and relations of number and led to clear and concise use of language." Miss Susan Blow.

"Almost invariably children show a strong tendency to cut out things in paper, to make, to build—a propensity which, if duly encouraged and directed, will not only prepare the way for scientific conceptions, but will develop those powers of manipulation in which most people are so deficient." Herbert Spencer.

Drawing.—You may bring to the class to-morrow the different squares mentioned above, drawn on paper. You may draw and measure for a while, but do not measure those you bring to the class.

"What we try to represent we begin to understand."

Froebel.

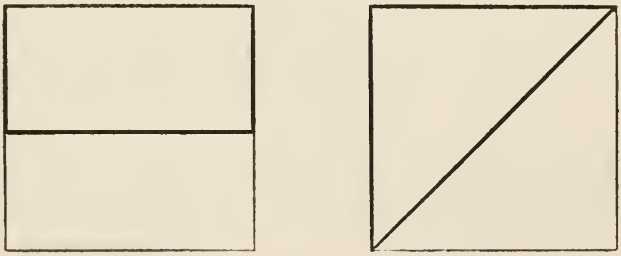
Remark: The pupils should be encouraged to cut and measure and to draw and measure many squares. Practice in this trains the hand and eye. Very little skill will be required by pupils who cut and draw only one square of each dimension given.

These exercises may seem very simple, but neither a boy of five nor a man of fifty can cut or draw a four-inch square who has not been trained to observe.

Upon our perceptions of form and of extent depends the correctness of our ideas of objects and consequently our power of giving true descriptions of things, of their location, their size, the relations of one part to another, etc., etc.

Give each pupil two four-inch squares. Give direction for folding the squares into an oblong rectangle. Place the rectangle on one side of the desk. Take the other square and place it so that one of its edges will extend in the same direction as the front edge of the desk. Fold the square so that the right-hand point in front will fall upon or coincide with the left-hand point at the back. Crease the paper. Place the two folded figures near each other for comparison.

COMPARISON OF THE OBLONG RECTANGLE WITH THE TRIANGLE.



"The mistake often made is that of supposing a pupil is learning how to *observe*, when he is merely listening to what his teacher *tells* him to *remember* about an object he may be looking at."

1. Find likenesses.
2. Find differences.
3. Find triangles in room.*
4. Recall forms that are triangular in shape. *Example:* The gables of some houses are triangular. The lateral surfaces of pyramids are triangles.
5. In going to and from school or at home you may find five objects that have triangular surfaces, and you may tell me the names of the objects that have these surfaces.
6. What are the names of the objects that you found having square surfaces?
7. What are the names of those that you found having oblong rectangular surfaces?

QUESTIONS—LIKENESSES.

SUGGESTION: Have pupils write the answers to the following questions in complete sentences. In the recitation have answers read, and let pupils criticise one another's statements.

1. Each form is what?

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*REMARK: Finding forms of the same general shape as those taken as types, is of the highest importance. Unless this is done, pupils are not learning to pass from the particular to the general. They are not taught to see many things through the one, and the impression they gain is that the particular forms observed are the only forms of this kind. Unless that which the pupil observes aids him in interpreting something else, it is of no value to him. Certain things are taught that through them other things may be seen. Pupils should not be trained to see for the sake of the seeing, but that they may have the power to see. How different the world appears to a child who sees form in everything from what it does to one who sees no definite form in anything, and to whom all is in a state of confusion. Teaching is leading pupils to discover the unity of things.

Ans. Each form is a surface.

2. Each surface is bounded by what?

3. The two short lines of the triangle are equal to what?

4. The sum of the two short lines of the oblong rectangle is equal to what?

5. The sum of the two acute angles of the triangle is equal to what?

6. Each form has, at least, one pair of what kind of lines?

7. Each form has, at least, one angle of what kind?

8. The area of the rectangle is equal to what part of the four-inch square?

9. The area of the triangle is equal to what part of the four-inch square?

10. The area of the triangle is equal to what?

11. Without observing the forms think of all the likenesses you can.

DIFFERENCES.

Write answers in complete sentences.

1. How many lines are there in the boundary of each surface?

2. Each surface has how many pairs of adjacent lines?

3. Eacn surface has how many angles or differences in direction?

4. Each has how many right angles?

5. Each has how many acute angles?

Find the right angle of the triangle.

Find the line opposite the right angle of the triangle. The line opposite the right angle of a right triangle is the hypotenuse of the right triangle.

6. The hypotenuse of a right angle is opposite what?

7. What is opposite the hypotenuse of a right triangle?

8. The hypotenuse of the right triangle is longer than what in the rectangle?

9. Is there any line opposite another line in the triangle?

10. How many pairs of opposite points are there in each form?

11. Are there any points in the triangle opposite any other points in it?

12. The sum of the angles of the rectangle is equal to how many right angles?

13. The sum of the angles of the triangle is equal to how many right angles?

14. The sum of the angles of the triangle is equal to what part of the sum of the angles of the rectangle?

15. What is there in the triangle that is not found in the rectangle?

16. What is there in the rectangle that is not found in the triangle?

17 Without observing the forms think of all the differences you can?

18. Write the directions for folding the square into the triangle.

CUTTING.—Cut a rectangle 2 inches long and 1 inch wide; another 3 inches by 2 inches; and another 4 inches by 2 inches. Pin them together. Do not measure those you bring to the class.

Cut a triangle having all its sides equal, another having only two sides equal, another of which no two sides are equal, and a fourth having a right angle. Write in the first, equilateral triangle; in the second, isosceles triangle; in the third, scalene triangle; and in the fourth, right triangle.

Can you cut a triangle having two right angles?

Can you cut a triangle so that the sum of two sides of it shall be equal to the third side?

Drawing.— Practice trying to draw rectangles of the same dimensions as those you cut. Measure all that you draw except those that you draw to bring to the class.

Cutting.— Observe a window at your home and cut a piece of paper in the same shape, and so that the edges shall have the same relation to each other as the edges of the window.

Example: If the window is 6 feet high and 3 feet wide, cut, as nearly as you can, a piece of paper 6 inches long and 3 inches wide, or 6 half inches long and 3 half inches wide, or so that the length of the paper shall be twice its width.

Drawing.— Observe the window and draw it in proper proportion.

Measure the window and write the measure on the paper on which you made the drawing, stating: The window is —— feet and —— inches high, and —— feet and —— inches wide.

Give each pupil two four-inch squares.

Have one of the pupils give directions for placing a square and folding it into a triangle.

Unfold the paper.

1. Do you see the line made by creasing the paper?

2. What does this line connect? The line joining the opposite points of the square is a diagonal line.

3. What does a diagonal line join?

4. Show me opposite points of the blackboard.

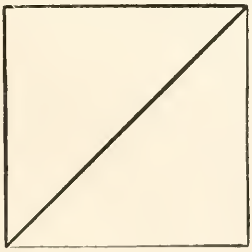
5. What is a line called which joins the opposite corners, or points, of a blackboard?

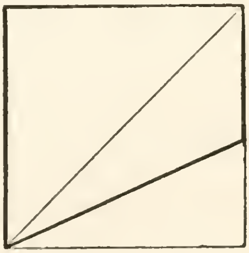
6. Show me what would be a diagonal line of the top of your desk.

7. What is a diagonal line of one of the surfaces of a pane of glass? Of one of the walls of the room?

Fold the square again into a triangle. Place it at one side and take the other square. Place the square so that one edge shall be parallel to the right edge of your desk. Fold the square so that the right-hand point in front will coincide with the left-hand point at the back. Crease the paper. Open the paper. Do you see the diagonal line? Fold the paper so that the front edge will coincide with the diagonal line. Crease the paper. Turn the paper over. The form you have folded is a trapezoid.

COMPARISON OF THE TRIANGLE WITH THE TRAPEZOID.





REMARK: The important part of the work suggested by this book consists in the simple exercises of finding and expressing the

likenesses and differences of the forms compared, and in finding similar forms in and outside of the school-room. These are the exercises that will foster habits of observation, and be of the greatest educational value. At least nine-tenths of the time given to the study of form should consist, not in answering the questions of the book, but in discovering relations not suggested by the questions. Incessant questioning, on the part of teacher or text, fixes on the pupil the habit of waiting to be questioned, and, when this condition is induced, the pupil's thinking generally ends with the questioning. The teachers who will succeed in this work, are those that have the courage to wait, and who can make their practice harmonize with the theory that it is what the pupil does for himself that educates him.

1. Find likenesses.
2. Find differences.
3. Find trapezoids in the room.*
4. Find, at least, five trapezoids in going to and from school or at home.

Remark: Trapezoids can be found where building is being done. Parts of the roofs of many houses are in this shape.

5. What are the names of the five objects which you found having triangular surfaces?

Write the answers to the following questions:

1. Each form has, at least, one pair of what kind of lines?
2. The difference in direction of two perpendicular lines is what kind of an angle?
3. Each form has at least one—angle and one—angle.
4. What is the name of the line opposite the right angle in the triangle?
5. The hypotenuse of the triangle is equal to what line of the trapezoid?

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*Suggestion: If at any time forms are being compared which can not be found in the room, the teacher should have several of these forms drawn on blackboard for the children to discover when looking for forms.

6. The sum of the two equal lines of the trapezoid and of the diagonal line is equal to what in the triangle?

7. To what is the sum of the two acute angles of the triangle equal?

8. The sum of the angles of the triangle is equal to what in the trapezoid?

9. Can you find two equal lines and two equal angles in the triangle? Which are they?

10. Can you find two equal lines and two equal angles in the trapezoid? Which are they?

DIFFERENCES.

1. Which figure has the greater surface, or area?

2. How many lines bound each form?

3. How many angles has each?

4. How many right, acute, and obtuse angles has each?

5. One of the acute angles of the triangle is equal to what part of one of the right angles of the trapezoid?

6. The acute angle of the trapezoid is greater than what in the triangle?

7. The acute angle of the trapezoid is greater than what part of the right angle of the triangle?

8. The longer of the two lines forming the obtuse angle of the trapezoid is shorter than what line in the triangle, and longer than what line?

9. An acute angle of the triangle is less than one-half of what angle of the trapezoid? How do you know?

10. The right angle in the triangle is greater than one-half of what angle in the trapezoid? Why?

11. If a right angle were equal to one-half of the obtuse angle of the trapezoid, to what, at least, would the obtuse angle be equal?

12. Do any of the lines of the triangle extend in the same direction? Any of the lines of the trapezoid? What, then, is true of the trapezoid that is not true of the triangle?

13. The sum of the lines of the triangle is less than what?

14. The shortest line of the trapezoid is greater than one-half of what, and less than one-half of what, in the triangle?

15. The sum of the two acute angles of the triangle is greater than what in the trapezoid, and less than what in it?

16. Which figure has opposite points and lines? How many pairs of each?

17. There are no pairs of opposite points or lines in which figure?

18. If both pairs of opposite points of the trapezoid were joined by lines, how many diagonal lines would it have?

19. Are there any diagonal lines in the triangle? Why not?

20. Write the direction for folding the square into a trapezoid.

FINDING FORMS MADE BY FOLDING.

Give each pupil a four-inch square. Place it for folding.

Fold the square so that the right-hand point in front will coincide with the left-hand point at the back.

Crease. Open the paper. Fold the square so that

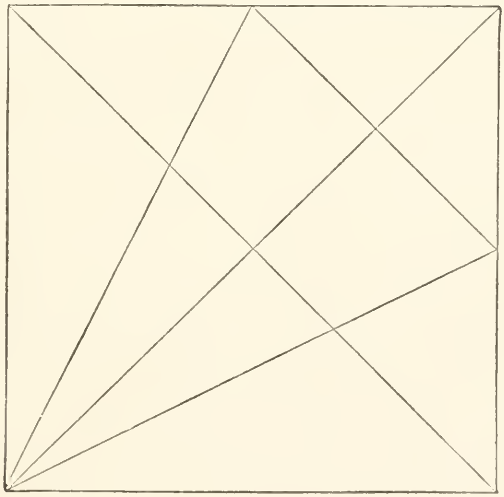
the left-hand point in front will fall upon the right-hand point at the back.

Crease Open the paper. Fold the paper so that the front edge will fall upon one of the diagonals.

Crease. Fold the left-hand edge so that it will coincide with the same diagonal.

Crease. Fold the paper so that the right-hand point at the back will fall upon a diagonal, and so that an isosceles triangle will be formed.

Crease paper carefully. Unfold the paper so that you will have the square again. Observe the forms made by the creased lines.



1. Observe the figure. How many triangles, each having a right angle, can you find?

2. How many triangles having two sides equal can you find?

3. How many triangles can you find having no two lines equal.

4. How many trapezoids can you find?

5. How many different figures have you found in the square?

Give each pupil two four-inch squares. Have some pupil give direction for placing the square and folding it into a trapezoid.

Take the other square. Fold it so that the right-hand point in front will fall upon or coincide with the left-hand point at the back. Crease the paper. Open it. Fold the paper so that the edge in front will coincide with the diagonal. Crease. Fold the paper so that the edge at the back will coincide with the diagonal. Crease. Turn the paper over. This form is called a rhomboid.

OBLIQUE ANGLES AND LINES.

Obtuse and acute angles are called oblique angles.

1. How many oblique angles are there in the rhomboid? Find oblique angles in the room.

2. What angles are called oblique angles?

3. Are there any oblique angles in the square?

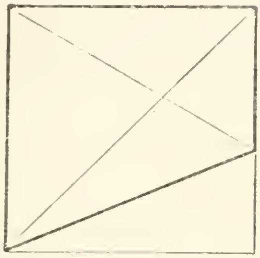
4. Are there any oblique angles in the trapezoid? How many?

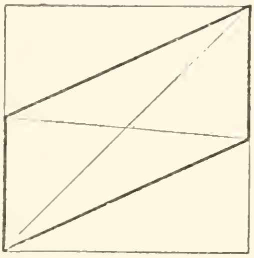
5. The lines forming either acute angles or obtuse angles are called oblique lines. How many pairs of oblique lines are there in the rhomboid?

6. Find oblique lines in the room. Are any of the lines of a rectangle oblique lines?

7. What kind of angles are formed by oblique lines?

COMPARISON OF THE TRAPEZOID WITH THE RHOMBOID.





1. Find likenesses.
2. Find differences.
3. Find rhomboids in room.
4. Find, at least, five rhomboids, and report the names of the objects that have surfaces of this shape.
5. What objects did you find having a surface or surfaces in the shape of a trapezoid?

QUESTIONS—LIKENESSES.

Write answers.

1. Each surface is bounded by how many lines and has how many angles?
2. How many pairs of opposite points and lines are there in each?
3. How many pairs of adjacent lines has each?
4. How many diagonal lines can each have?
5. The longer diagonal of the rhomboid is equal to what in the trapezoid?
6. The obtuse angle of the trapezoid is equal to what in the rhomboid?
7. One of the acute angles in the rhomboid is equal to what in the trapezoid?
8. The shortest lines of the trapezoid is equal to what in the rhomboid?

9. The longest boundary line of the trapezoid is equal to what in the rhomboid?

10. The sum, of the longer diagonal line, the shortest line and the longest line of the trapezoid is equal to what in the rhomboid?

11. There is, at least, one pair of opposite lines extending how in each form?

12. That part of the trapezoid bounded by its longest line, shortest line and the longer diagonal line is equal to what in the rhomboid?

13. Can you find two equal angles and two equal lines in each? What or which are they?

DIFFERENCES.

1. How many different angles are there in each form?

2. What kind of lines are found in the trapezoid that are not found in the rhomboid?

3. How many pairs of parallel lines are there in each?

4. One of the two equal lines of the trapezoid is longer than what and shorter than what in the rhomboid?

5. The shorter diagonal of the trapezoid is longer than what in the rhomboid and shorter than what in the same figure?

6. Which form has the greater area?

7. Write the directions for folding the rhomboid.

CUTTING.—Observe and cut two straight line figures in proper proportion, such as doors, windows, floors, walls, blackboards, tops of tables, sides and ends

of boxes, covers of books, etc., etc. Measure the two dimensions of the objects observed and write the measure on the paper cut.

Drawing.—Observe and draw two straight line figures. Write the measures of the two dimensions on the drawing.

Can you draw a trapezoid, having only one right angle?

Give each pupil two squares.

Have some pupil give directions for placing and folding a square into a rhomboid.

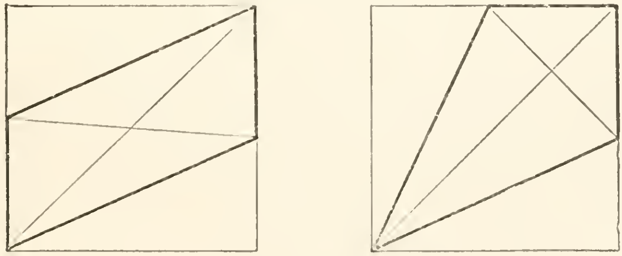
Directions for folding the square into a trapezium:

Place a square so that its right edge will extend in the same direction as the right edge of the desk.

Fold the square so that the right point in front will fall upon the left point at the back. Crease.

Open the paper. Fold the paper again so that the front edge will lie along the diagonal line. Crease. Fold the paper so that the left edge will lie along the same diagonal. Crease. Turn the paper over and place it for comparison with the rhomboid.

COMPARISON OF THE RHOMBOID WITH THE TRAPEZIUM.



1. Find likenesses.

2. Find differences.

3. Find trapeziums in room.

4. Find five objects that have surfaces or a surface in the shape of a trapezium. Any four-sided figure having none of its edges parallel is a trapezium.

5. What objects did you find having a surface or surfaces in the shape of a rhomboid?

QUESTIONS—LIKENESSES.

Write answers to the following questions:

1. The two long lines of the trapezium are equal to what in the rhomboid?

2. The two short lines of the trapezium are equal to what?

3. What is true of one of the diagonals of each?

4. Into what does the longer diagonal of each figure divide it?

5. The area of half of the trapezium is equal to the area of what?

6. The area of the rhomboid equals what?

7. Each form has how many obtuse angles?

8. The sum of the right angle and the acute angle of the trapezium equals what in the rhomboid?

9. If the two obtuse angles of each form are equal, and if the sum of the right angle and acute angle of the trapezium equal the sum of the two acute angles of the rhomboid, to what is the sum of the angles of the trapezium equal?

DIFFERENCES.

1. What is true of two adjacent lines of the tra-

pezium that is not true of any of the adjacent lines of the rhomboid?

2. What is true of the two short lines of the trapezium that is not true of any of the lines of the rhomboid?

3. What kind of an angle is found in the trapezium that is not found in the rhomboid?

4. What is true of the opposite lines of the rhomboid that is not true of the opposite lines of the trapezium?

5. Are all trapeziums kite-shaped?

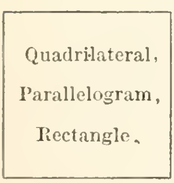
6. Must a trapezium have a right angle?

7. Can a trapezium have two right angles?

8. Can a trapezium have two adjacent right angles?

9. Write the direction for folding a square into a trapezium.

CLASSIFICATION OF FORMS.



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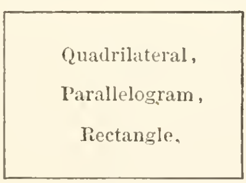

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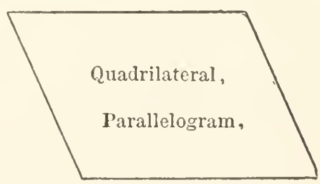

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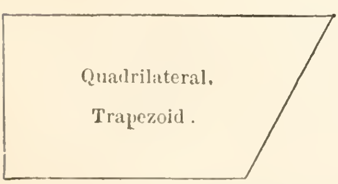
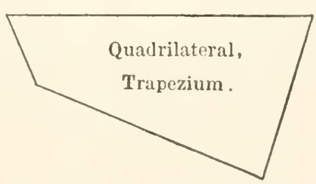




1. In what respects are all these figures alike?

2. What general name can be used in speaking of any of the above forms?

3. In what are *a*, *b* and *c* alike?

4. By what common name can you speak of *a*, *b* and *c*?

5. What common name have *a* and *b*?

6. Are all quadrilaterals parallelograms? Are all parallelograms quadrilaterals?

7. Are all rectangles parallelograms? Are all parallelograms rectangles?

8. Are all squares rectangles? Are all rectangles squares? Why not?

9. What is true of an oblong rectangle that is not true of the square? What is true of a square that is not true of an oblong rectangle?

10. In what respects are the square, oblong rectangle and trapezoid alike?

11. How many pairs of parallel lines has a square, an oblong rectangle, and a trapezoid, respectively?

12. A parallelogram is a quadrilateral whose opposite sides extend in the same direction, or are parallel.

13. Why is a square a parallelogram?

14. Is a rhomboid a parallelogram? Why?

15. Can you draw a quadrilateral that is not a parallelogram?

16. Can you draw a quadrilateral that is neither a parallelogram nor a trapezium? What is its name?

17. Is a trapezoid a quadrilateral? Is a trapezoid a parallelogram? Are both pairs of opposite sides in the trapezoid parallel?

18. Can you find five angles, or differences in

direction, in the trapezoid? Can you find six in a trapezium?

19. Draw a figure having at least one right angle and four equal sides. What is its name?

20. Draw a figure having at least one pair of perpendicular lines and four equal sides. What is it?

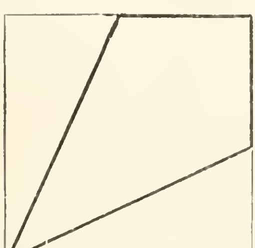
TRAPEZIUM AND SCALENE TRIANGLE.

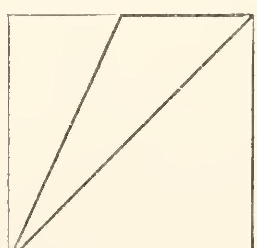
Give each pupil two squares. Have some pupil give the directions for folding the trapezium. Place the trapezium at one side, and have another pupil give the directions for folding the other square into a trapezium.

Observe the trapezium. Do you see an obtuse angle? Do you see the point at which the two lines forming the obtuse angle meet? This point is the vertex of the angle. How many angles has the trapezium? How many vertices?

Directions for folding the trapezium into a scalene triangle:

Fold the trapezium so that the vertex of one of the obtuse angles will fall upon the vertex of the other obtuse angle. Crease the paper. Write a comparison of the scalene triangle with the trapezium.





1. Give likenesses.

2. Give differences.

3. Find scalene triangles in the room.

4. Write the names of five objects that have surfaces in the shape of trapeziums.

5. Find five scalene triangles at home or in going to and from school.

6. Write the directions for folding the square into the scalene triangle.

Have pupils cut and bring to the class to-morrow one of each of the forms compared. Have them cut trapezoids and trapeziums differing in shape from the ones that have been compared. When the forms are brought in, mix them and have pupils give name and description of the form selected. *Example:* This quadrilateral is a trapezoid as it has only two sides parallel. This quadrilateral is a rectangle as its angles are right angles. It is an oblong rectangle, because it is longer than it is wide.

Have pupils find forms in the collection, from descriptions given by other pupils. Have just enough of a description given to enable the one who selects to find the form. Let the pupil who gives the description name the one who is to find the form. *Examples:*

1. Find a quadrilateral having two pairs of parallel lines. What is its name? Find another having a different name but having two pairs of parallel lines. What is its name?

2. Find a quadrilateral having only two lines parallel. What is its name?

3. Find a parallelogram of which any two of its adjacent lines are equal. What is its name?

4. Find a rectangle that is longer than it is wide. What is its name?

REMARK: These exercises will lead to close observation, which is the basis of correct expression and exact reasoning.

THE SCALENE TRIANGLE AND RHOMBUS.

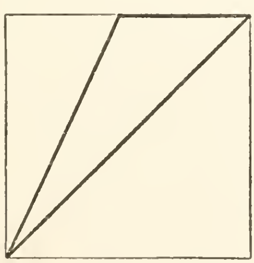
Give pupils two squares.

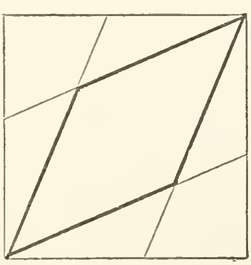
Have a pupil give the direction for folding a square into a scalene triangle.

Have another pupil give the direction for folding a square into a trapezium.

Direction for folding the trapezium into a rhombus: Fold the trapezium so that one of its short lines will coincide with the diagonal. Crease. Fold the paper so that the other short line will fall upon the diagonal. Crease.

Write a comparison of the scalene triangle with the rhombus:





1. Give likenesses.
2. Give differences.
3. Find rhombuses in room.
4. What objects did you find having a scalene triangle for a surface?

5. Find five objects that have a surface or surfaces that are rhombuses, and give the names of the objects to-morrow.

6. Write the directions for folding the rhombus.

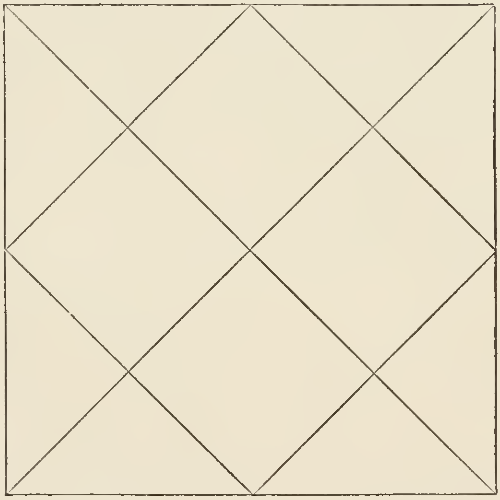
FINDING FORMS MADE BY FOLDING.

Give each pupil a square.

Fold the square so that the right-hand point in front will fall upon the left-hand point at the back.

Crease. Open the paper. Fold the paper so that the left-hand point in front will fall upon the right-hand point at the back.

Crease and open the paper. Fold each point to the center and crease. Open the paper.



How many squares, oblong rectangles, trapezoids and triangles can you find in the figure?

Draw each figure. Try to draw the figures the same size, and in the same proportion, as those you see in the square.

THE RHOMBUS AND THE ISOSCELES TRIANGLE.

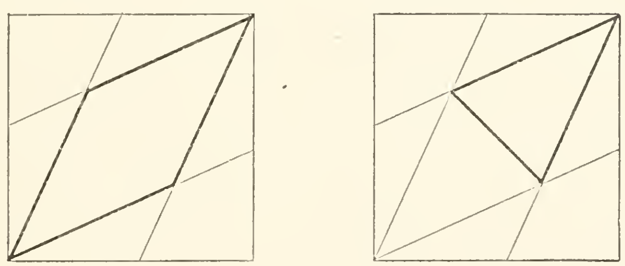
Give each pupil two squares.

Have pupils give directions for folding each square into a rhombus.

The following are the directions for folding the rhombus into an isosceles triangle:

Fold the rhombus so that the vertex of one of the acute angles will fall upon the vertex of the other acute angle. Crease the paper.

Write a comparison of the rhombus with the isosceles triangle.



1. Find likenesses.

2. Find differences.

3. Find isosceles triangles in room.

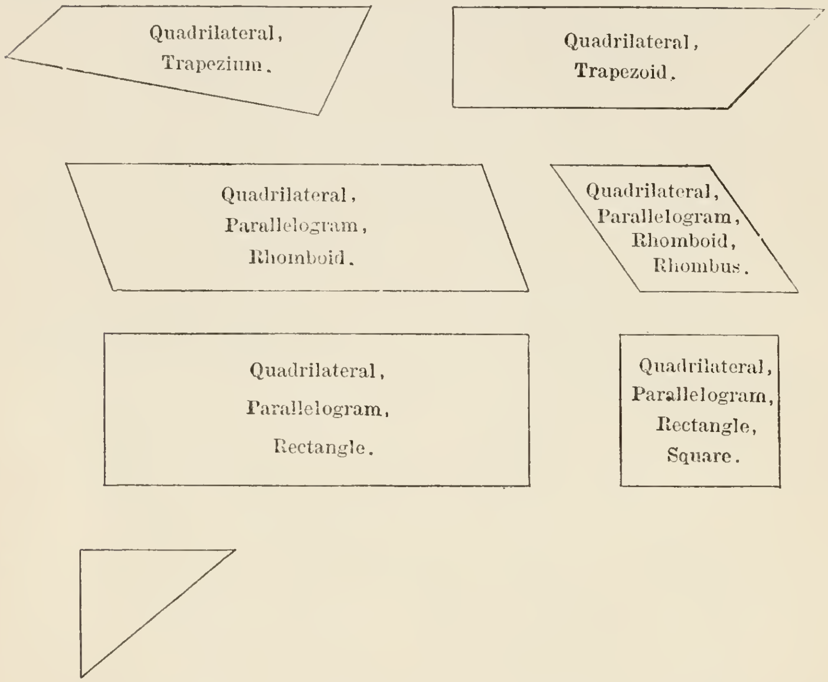
4. Write the names of the objects which you found having a surface or surfaces like a rhombus.

5. Find five objects which have a surface in the shape of an isosceles triangle.

6. Write the direction for folding the isosceles triangle.

CLASSIFICATION OF FORMS.

SUGGESTION: Omit work on classification of forms until third year.



Write such a description of each of the above forms that the person reading it can select the form described. Make the descriptions as short as you can. *Example:* It is a quadrilateral with no sides parallel.

What is the name of the figure described?

1. What is wrong in the following description of a square?

A square is a figure bounded by four equal lines.

Of how many of the forms is the description true?

2. What is wrong?

A square is a figure bounded by four lines and having four right angles.

Of how many figures is the description true?

3. What is wrong?

A rhombus is an oblique-angled parallelogram.

How many figures does this describe?

4. What is wrong?

A rhombus is a figure having four equal lines.

How many figures are included in this description?

5. A quadrilateral is a plane figure bounded by four straight lines.

How many of the above figures are quadrilaterals?

6. A quadrilateral whose opposite lines are parallel is a parallelogram.

How many of the figures are parallelograms?

7. A rhomboid is a parallelogram whose angles are oblique angles.

How many of the figures are rhomboids?

Is a rhombus a rhomboid?

Are all rhomboids rhombuses?

8. A rectangle is a parallelogram whose angles are right angles.

How many of the figures are rectangles?

Is a square a rectangle?

Are all squares rectangles? Are all rectangles squares?

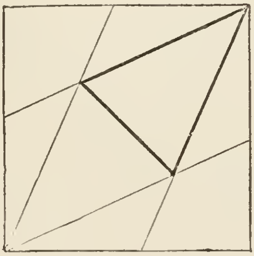
Call attention to the different figures cut out of paper, drawn on the blackboard, or found in the room, and have pupils tell what they are and define them. *Example:* The blackboard is a rectangular parallelogram because its opposite lines are parallel and its angles are right angles.

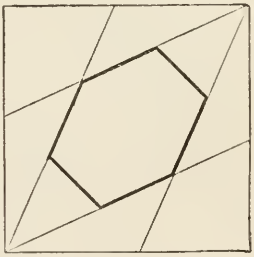
Suggestion: It would be well to have the pupils re-write the comparisons, condensing them as much as they can. In comparing the square with the rectangle, all the likenesses could be given in one or two sentences; as, each form is a surface having four lines, four points, four pairs of perpendicular lines, four pairs of adjacent lines, two pairs of opposite lines, two pairs of parallel lines, and four right angles.

THE ISOSCELES TRIANGLE AND THE HEXAGON.

Give each pupil two 4-inch squares. Have a pupil give the directions for folding the square into an isosceles triangle. Have another pupil give the directions for folding the other square into a rhombus.

The following are the directions for folding the rhombus into a hexagon: Fold the rhombus so that the vertex of one of the acute angles will fall upon the center of the rhombus. Crease. Fold the paper so that the vertex of the other acute angle will coincide with the same point. Crease. Compare the isosceles triangle with the hexagon. Write.





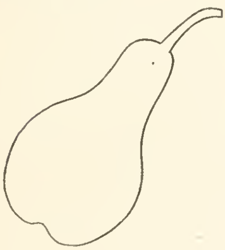
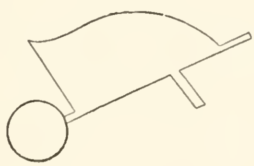
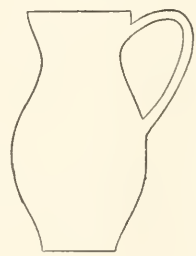
1. Find likenesses.
2. Find differences.
3. Find hexagons in room.
4. Write the names of the objects in which you found isosceles triangles.

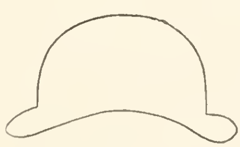
CUTTING AND DRAWING.

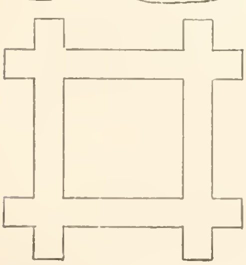
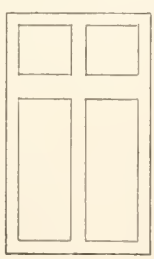
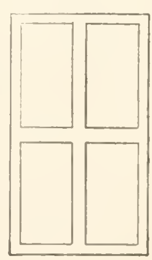
Have pupils observe and cut in outline forms in art and of animals, inkstands, flower-pots, buckets, baskets, ladders, cups, teapots, cows, horses, pigs, etc., etc. Preserve pupils' work by pasting it on cardboard.

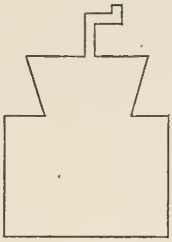
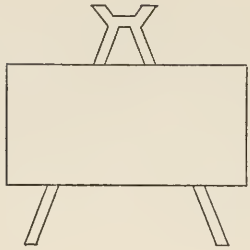
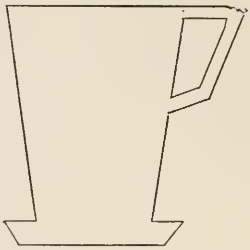
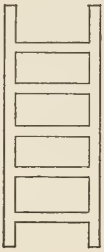
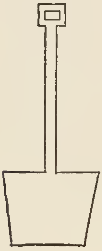
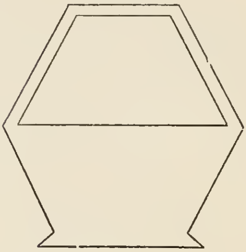
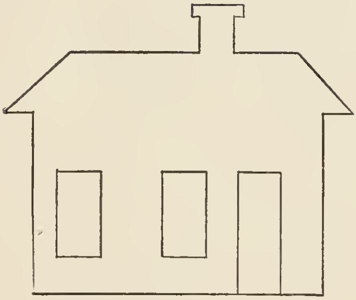
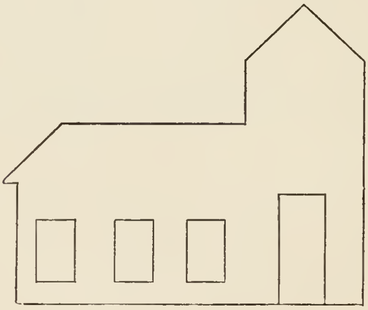
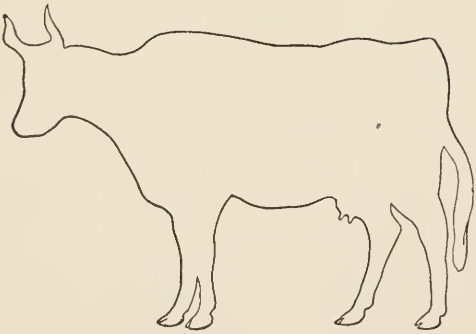
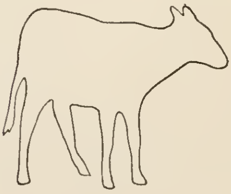
Have pupils observe and draw in outline the same forms that they cut.

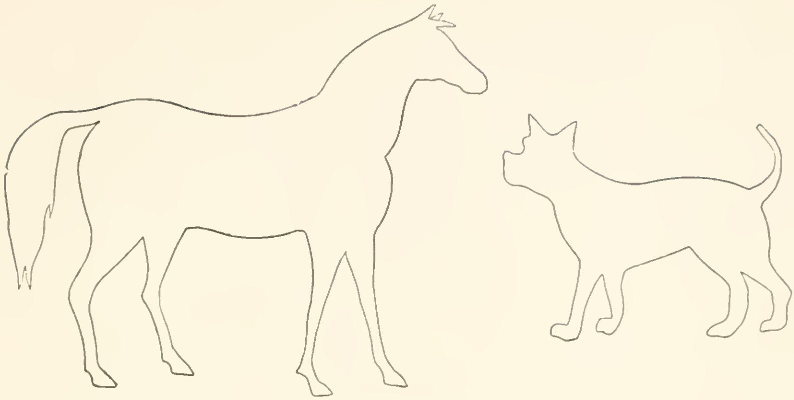
The following are a few examples of the many things that may be observed and cut in outline and observed and drawn. The things themselves, and not these diagrams, are to be observed in the cutting and drawing.





POINTS AND LINES.

POINTS AND LINES.

QUESTIONS.

SUGGESTION: Omit work on points, lines and surfaces with first, second, and third year pupils. Omit with fourth year pupils until after the comparisons of the solids.

1. If a point be moved, its path is what? What, then, is the path of a moving point?

2. The path of a point that does not change its direction is what kind of a line? What, then, is a straight line?

3. If a point is moved so as to change its direction continually, what kind of a line will it make, or generate? What, then, is a curved line?

4. If two points are moved in the same direction, what will be true of the lines generated? How may parallel lines be generated?

5. How far must a point be moved to generate a line?

1. Does a point have length?
2. Would two or more points have length?
3. What dimension does a line have?

4. Is a part of a line a line?

5. Is a point a part of a line? Why not?

6. Would a number of points make, or constitute, a line? Why not?

7. If you know the position of two points in a straight line, can you tell the position and direction of the line?

POINTS, LINES AND SURFACES.

QUESTIONS.

1. What will a moving line make, or generate?

2. How can a line be moved so as not to make, or generate, a surface?

3. When a line is moved so as to generate a surface, what will the extremities of the line generate?

4. How far must a line be moved to generate a surface?

5. If a straight line is moved so as to continue parallel to its first position, what is true of the length of the lines that its extremities generate? What is true of the direction of the two lines generated by its extremities?

6. What two thing are true of the lines generated by the extremities of a line moved so as to continue parallel to its first position?

7. If a straight line is moved so that its extremities generate curved lines, what will the line itself generate? How must a line be moved so as to generate a curved surface?

8. Could a curved surface be generated by a straight line, if one of its extremities did not change its position?

9. If a straight line is moved so that its extremities generate straight lines, what kind of a surface does the line itself generate?

10. What is the name of the figure generated by moving a straight line so that its extremities generate equal straight lines?

11. How may a parallelogram be generated?

12. If a straight line is moved so that its extremities generate equal straight lines, which are perpendicular to the given line in its first position, what figure is generated by the line itself?

13. How may a rectangle be generated?

14. How may a square be generated?

15. How may a square be generated, beginning with a point?

16. Will the extremities of a straight line generate straight lines unless both extremities are moved at the same rate?

17. Can a trapezoid be generated by a straight line?

1. How many dimensions has a line?

2. How many dimensions has a surface?

3. How many dimensions has a part of a surface?

4. Is a line a part of a surface?

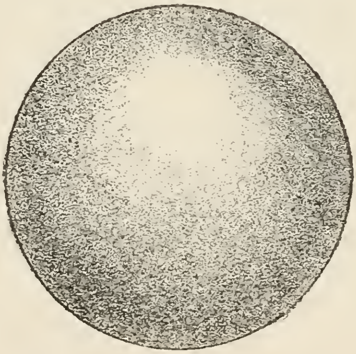
5. Has a line width? Would two or more lines have width? Would a number of lines make, or constitute a surface?

Part Second.

SOLIDS.

THE SPHERE.

Have pupils make a sphere of clay. If the clay is not at hand, place a sphere, or ball of some kind, before them for observation.



1. Have you seen other forms of this shape in or about the school-room?

2. What are the names of the objects that you have observed in the shape of a sphere?

3. Name five objects that you have seen at home or in other places, in the shape of a sphere. *Example:* I have seen some grapes in the shape of a sphere.

4. The sphere is limited, or bounded, by what kind of surface?

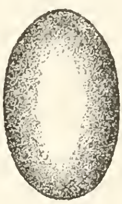
5. Find objects in the room that are limited or partially limited by a curved surface, and tell the name of each.

6. Hand me, to-morrow, the names of ten objects that you have observed that have a curved surface.

DRAWING.—1. Observe and draw some object in the shape of a sphere.

2. Draw, from memory, another sphere two inches in diameter. Measure.

AN OBLATE SPHEROID.



1. Observe and model out of clay an oblate spheroid.*

2. Have you seen any oblate spheroids at school? *Example*: Some door-knobs are oblate spheroids.

3. What are the names of five oblate spheroids that you have seen at home or in other places?

4. To-morrow, hand me, on paper, the names of five oblate spheroids.

DRAWING.—1. Observe and draw an oblate spheroid.

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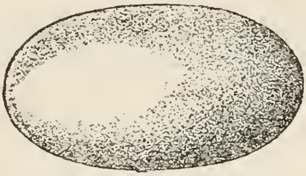
* NOTE: If you do not have an oblate spheroid, use a door-knob, a field turnip, or some other common object in the shape of an oblate spheroid, for a model.

2. Draw another oblate spheroid with one of its longest diameters two inches, and its shortest, one inch. Measure.

A PROLATE SPHEROID.



1. Observe and make out of clay a prolate spheroid. A potato in the shape of a prolate spheroid will do for a model.

2. What objects have you seen in the shape of a prolate spheroid.

3. Hand in, to-morrow, on paper, the names of five prolate spheroids.

4. Does the entire surface of a prolate spheroid curve uniformly?

5. What can you say of a curved surface of the sphere?

Drawing.—Observe and draw some object in the shape of a prolate spheroid.

CONVEX AND CONCAVE.

1. The surface of a drop of water is convex. Find five convex surfaces in the room.

2. Recall five objects that have convex surfaces.

3. Observe objects and write five sentences using in each the term convex.

4. A surface that is hollow and curved or rounded

is concave. *Example:* The hollow of the hand or the inside surface of a watch crystal is concave.

5. Find five concave surfaces in the room.

6. Observe objects and write five sentences using in each the term concave.

7. Write the names of five objects that have both concave and convex surfaces.

A curved Line

1. Find five curved lines, or edges, in the room.

2. Recall five objects that you have seen that have curved edges, or lines. *Example:* The edge of the tire of a wagon-wheel is curved.

COMPARISON OF SOLIDS.*

1. Have each pupil make a sphere of clay.

2. Make another sphere of the same size as the first. Use a small wire and cut one of the spheres into hemispheres.

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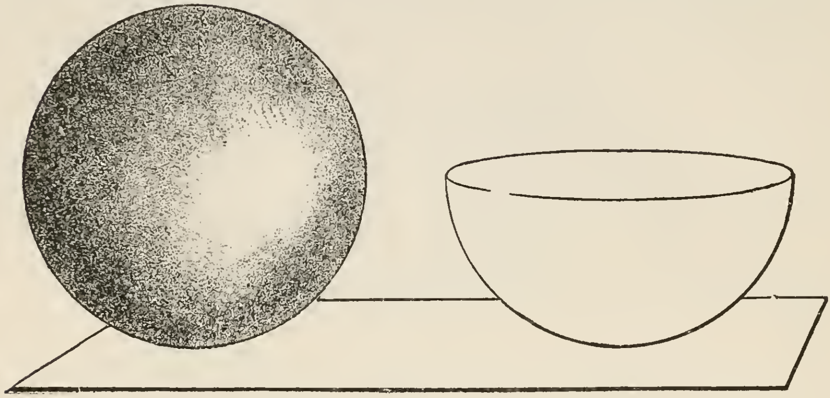
*REMARK: If there is no clay provided for making the solids to be used in the comparisons, get forms made of wood and use them. Do not use drawings as the basis for the comparisons. Drawings are poor substitutes for concrete representations of the forms to be compared.

COMPARISON OF THE SPHERE WITH THE HEMISPHERE.



1. Find likenesses.
2. Find differences.
3. Recall objects in the shape of a hemisphere,

Example: Some birds's nest are like a hemisphere.

DRAWING.

Cut out of pasteboard a circle having a diameter of 1 foot.

1. Place the circle on some object whose top is on a level with the eyes of the pupils, so that the edge of the circle will appear to be a horizontal straight line. Direct pupils to observe the circle and draw just what they see.

2. Place the circle so that its edge will appear to be a vertical, or upright, line to the pupils in the middle row. Direct pupils to observe the circle and represent what they see.

REMARK: If pupils draw what they see, the middle row will draw straight lines only. Each of the other pupils will draw an ellipse. The pupils who will see an ellipse approaching nearest the circle will be those in the front seat of the right and left-hand rows.

3. Place the circle so that the right or the left-hand row will see the entire circle. Direct pupils to draw what they see.

REMARK: If pupils are in the habit of determining the proportion of figures by holding the pencil at arm's length and measuring with the eye, it would be well to make the measurements after and not before the drawing is made.

Cut out of pasteboard a large square and other plane figures, and place them in different positions before the class for drawing. If pupils can represent just what they see of plane figures, it will aid them greatly in drawing solids correctly.

Draw a sphere and a hemisphere.

1. Write likenesses.
2. Write differences.

QUESTIONS ON THE SPHERE AND HEMISPHERE.

LIKENESSES.

1. Each form has what kind of a surface?

2. What kind of a curved surface does each have?

3. What is true of all the lines extending from the center of the sphere to its surface? One of these lines is called a radius; two or more of them radii. What is true of the radii of the sphere?

4. Make a point in the center of the plane surface of the hemisphere. What is true of all the straight lines extending from the center of the plane surface of the hemisphere to its curved surface?

5. What lines are equal in both the sphere and hemisphere?

6. What is true of the center of the sphere and of the center of the plane surface of the hemisphere?

7. In what respect are the circumference of the plane surface of the hemisphere and the curved surface of the sphere alike? Do they each curve uniformly?

8. A straight line drawn from the center of the plane surface of the hemisphere to its circumference is called a radius. What is true of the radii of the circle limiting the hemisphere?

9. How does a radius of the circle compare with a radius of the sphere?

10. Straight lines passing through the center of the sphere and terminating in its surface are called diameters of the sphere, and straight lines passing through the center of the plane surface of the hemisphere and terminating in its circumference are diameters of the circle. What is true of the diameters of the sphere and of the plane surface of the hemisphere?

DIFFERENCES.

1. Each solid is limited by how many surfaces?

2. The hemisphere is limited by what kind of a surface that is not found in the sphere?

3. What is the name of the plane surface of the hemisphere?

4. Find other circles in the room.

5. Recall five objects that are limited by circles. *Example:* The bottoms of some buckets are circles.

6. What kind of a line did you find in the hemisphere?

7. A hemisphere is equal to what part of a sphere?

8. The curved surface of the hemisphere is equal to what part of the curved surface of the sphere?

9. Can you draw a straight line on the surface of each solid?

10. Can you draw a curved line on the plane surface of the hemisphere?

11. What two things are found in the hemisphere that are not found in the sphere?

12. Describe a sphere, without naming it, so that, by the description it may be selected from a collection of different solids.

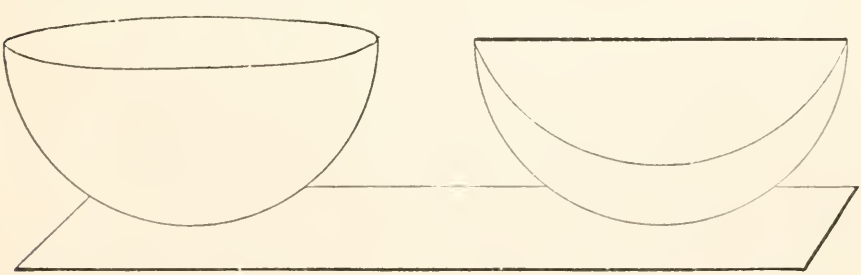
13. Recall the things that you have learned about the sphere and the hemisphere.

14. Draw the sphere and the hemisphere, and write a comparison of the two forms. In the comparison use the terms uniformly, radius, radii, diameters, circle and circumference.

THE HEMISPHERE AND THE QUADRANT OF A SPHERE.

Make a sphere of clay. Separate the sphere into hemispheres. Separate one of the hemispheres into two similar and equal parts.

Compare the hemisphere with the quadrant, or quarter of the sphere.



1. Find likenesses.
2. Find differences.

3. What objects did you find in the shape of a hemisphere?

QUESTIONS—LIKENESSES.

1. The hemisphere and the quadrant of the sphere are bounded by what kind of surfaces?

2. Each has at least one —— line.

3. To what is the sum of the plane surfaces of the quadrant equal?

4. To what is the sum of the two curved lines of the quadrant equal?

5. What is the name of the line in the hemisphere of which each of the curved lines of the quadrant is equal to one-half.

DIFFERENCES.

1. Each solid is limited by how many surfaces?

2. The surfaces of each are limited by how many lines?

3. What kind of a line do you find in the quadrant that is not found in the hemisphere?

4. Besides the straight line, what else do you find in the quadrant that is not found in the hemisphere?

5. How many points do you find in the quadrant?

6. How many semi-circles limit the quadrant?

7. The curved surface of the quadrant equals what part of the curved surface of the hemisphere?

8. Is the curved surface of any quadrant of a sphere equal to one-half the curved surface of any hemisphere?

9. The difference in direction of two plane surfaces is a dihedral angle.

Find dihedral angles in the room.

When you point to a dihedral angle, say that the difference in direction — indicating the directions of the two planes by moving hand or pointer — of this plane and that plane is a dihedral angle.

REMARK: To show the direction of the planes the pointer should be moved in a line perpendicular to the intersection of the two planes.

10. Does the quadrant have a dihedral angle? Why?

11. How many things do you find in the quadrant that are not found in the hemisphere? In the hemisphere that are not found in the sphere?

Draw the hemisphere and the quadrant of the sphere.

Write a comparison of the hemisphere with the quadrant of the sphere.

1. Write likenesses.
2. Write differences.

THE QUADRANT AND THE EIGHTH OF A SPHERE.

Make a sphere of clay.

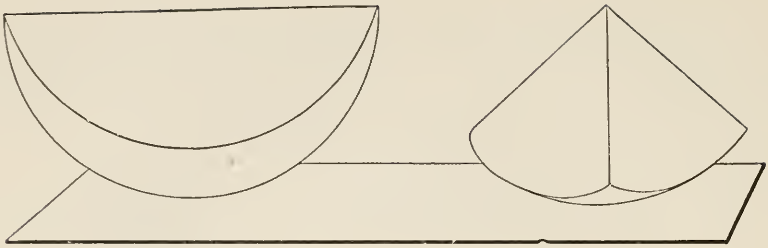
Separate the sphere into hemispheres.

Separate one of the hemispheres into a quadrant of a sphere.

Pass a wire through the middle point of the straight line of the quadrant and separate the quadrant into two equal parts.

What part of a sphere is one of the two equal parts of a quadrant of a sphere?

Compare the quadrant of a sphere with the eighth of a sphere.



1. Find likenesses.
2. Find differences.

QUESTIONS—LIKENESSES.

1. Each solid is limited by what kind of surfaces?

2. Each solid is limited by how many curved surfaces?

3. The surfaces of each solid are limited by what kind of lines?

4. The lines of each surface are limited by what?

5. The difference in direction of two plane surfaces is what kind of an angle?

6. Each surface has at least one —— angle.

7. Is the difference in direction of the two plane surfaces of the quadrant of the sphere equal to the difference in direction of two of the plane surfaces of the eighth of the sphere?

DIFFERENCES.

1. Each solid is limited by how many plane surfaces?

2. The surfaces of each solid are limited by how many straight lines?

3. The lines of each solid are limitea by how many points?

4. In the quadrant of the sphere, two points are the limit of how many lines?

5. In the eighth of the sphere, four points are the limit of how many lines?

6. One of the plane surfaces of the eighth of the sphere is equal to what part of one of the plane surfaces of the quadrant of the sphere?

7. The sum of the plane surfaces of the eighth of the sphere is equal to what part of the sum of the plane surfaces of the quadrant of the sphere?

8. One of the straight lines of the eighth of the sphere is equal to what part of the straight line of the quadrant of the sphere?

9. The straight line of the quadrant is equal to what part of the sum of the straight lines of the eighth of the sphere?

10. One of the curved lines of the eighth of the sphere is equal to what part of one of the curved lines of the quadrant?

11. The sum of the curved lines of the eighth of the sphere is equal to what part of the sum of the curved lines of the quadrant?

12. Each solid has how many dihedral angles?

13. The difference in direction of three or more plane surfaces is a solid angle. What does the eighth of a sphere have that the quadrant does not have?

14. Find solid angles in the room. *Example:* The difference in direction of the two walls and the ceiling of the room is a solid angle.

15. How many solid angles has a cube? How many dihedral angles?

16. What is the difference in direction of the three plane surfaces of the eighth of the sphere called?

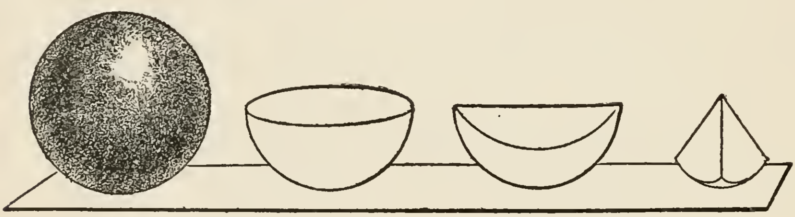
17. The straight lines of the eighth of the sphere extend in how many directions?

18. How many straight lines are necessary to represent, or show, a difference in direction?

19. The straight lines of the eighth of the sphere form how many angles?

20. How many pairs of perpendicular lines are there in the eighth of the sphere?

21. The eighth of the sphere has how many right angles?



22. What did you find in the hemisphere that is not found in the sphere?

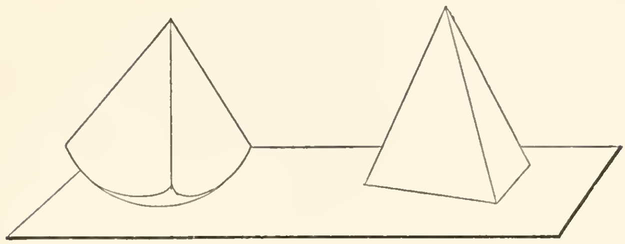
23. What did you find in the quadrant of the sphere that is not found in the hemisphere?

24. What was found in the eighth of the sphere that was not found in the quadrant of the sphere?

25. What was found in the eighth of the sphere that was not found in the sphere?

Observe and draw the quadrant of the sphere and the eighth of the sphere. Write a comparison of the quadrant of the sphere and the eighth of the sphere.

EIGHTH OF A SPHERE AND A TRIANGULAR PYRAMID.



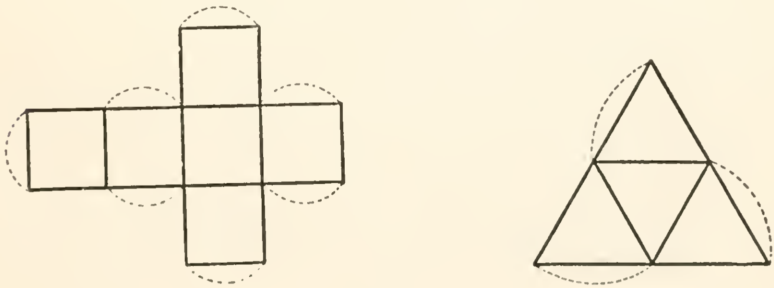
Make and compare the eighth of a sphere with the triangular pyramid.

1. Find likenesses.
2. Find differences.
3. Draw each and write a comparison.

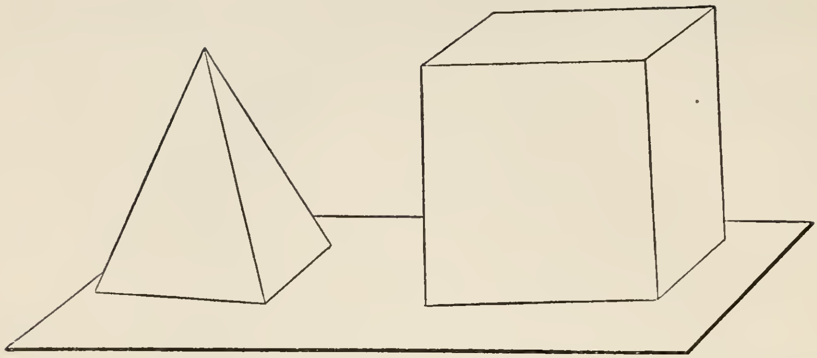
TWO-INCH CUBE AND THE TRIANGULAR PYRAMID.

Make models of the 2-inch cube and the triangular pyramid.

SUGGESTION: Draw on thick, tough cardboard the diagrams shown below. Cut the diagrams out of the cardboard and cut half through the edges of the surfaces that are joined. Then fold the figures into the solids to be compared. Fasten the adjacent surfaces of the solids by means of mucilage or paste.



Observe and compare the models you have made.

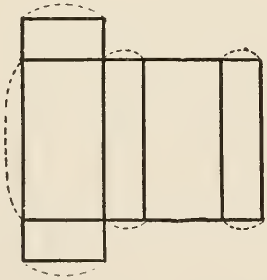


Draw the 2-inch cube and the triangular pyramid and write a comparison of them.

1. Likenesses.
2. Differences.
3. Write the names of five cubes.

From the following diagrams, models of the (1) quadrangular prism, (2) triangular prism, (3) hexagonal prism, (4) cylinder, (5) cone, (6) frustrum of a cone, (7) quadrangular pyramid, (8) tetrahedron, (9) octahedron, and (10) icosahedron can be developed.

A comparison of these forms will serve both as language exercises and as an introduction to the study of solid geometry.



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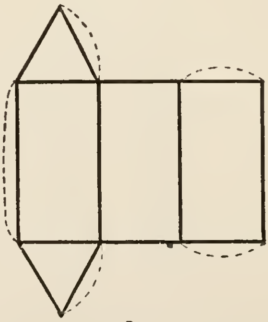

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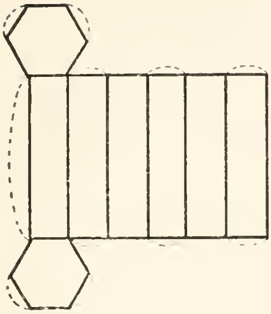

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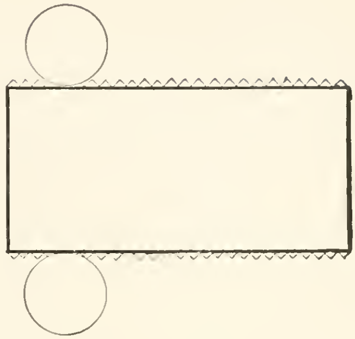

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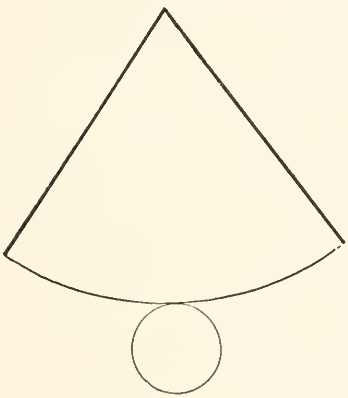

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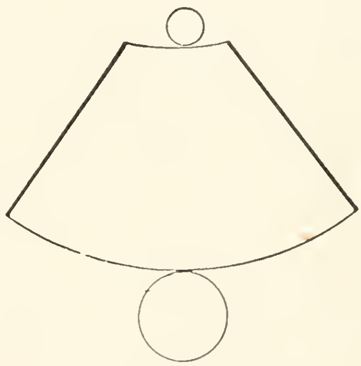

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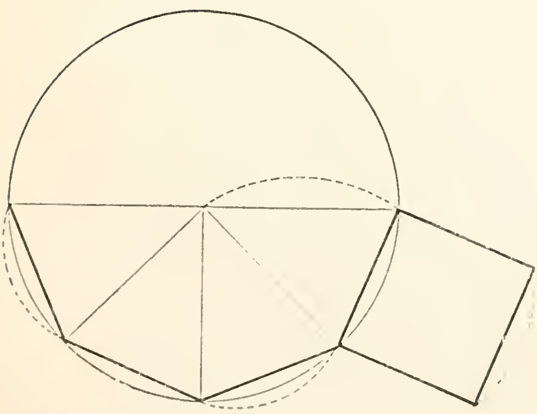

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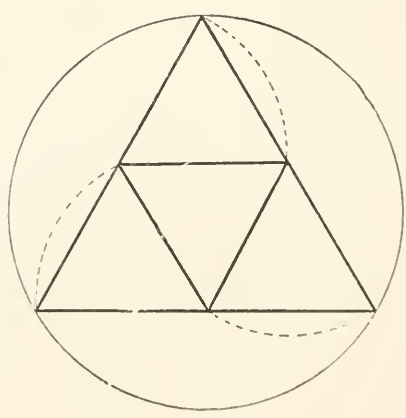

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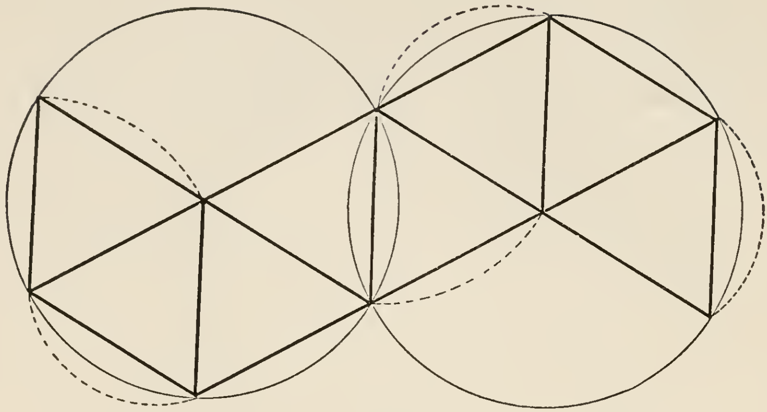

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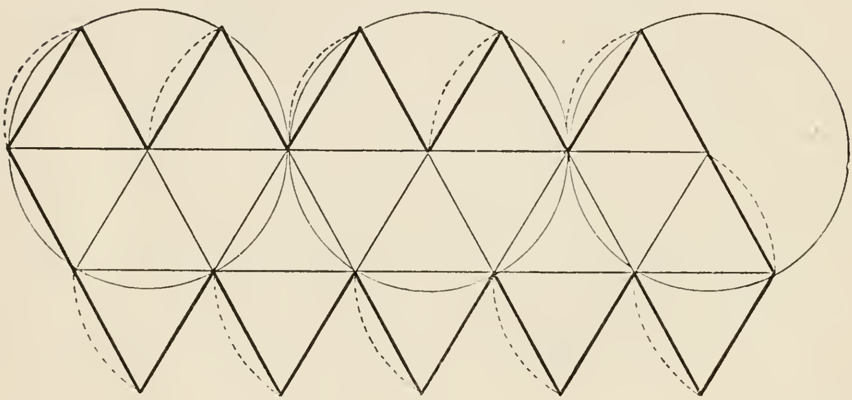

10

POINTS, LINES, SURFACES AND SOLIDS.

QUESTIONS.

SUGGESTION: Omit in the primary grades.

1. If a square is moved so that it continues parallel to its first position, and in one direction, what will its edges generate? What will the four points of the square generate? How do the lines generated by the four points compare as to length and direction? The edges of the square generate what kind of figures? Why? Do you know whether the parallelograms generated are rhomboids or rectangles? Why not? If

the edges generate rectangles, what kind of a solid does the square generate?

2. If a square is moved so that its edges generate surfaces which are perpendicular to the square in its first position, what kind of a solid will the square generate?

3. How far and in what manner must a square be moved to generate a cube?

4. How may a cube be generated, beginning with a point?

5. If a rectangle be moved about one of its edges as an axis, what will the extremities of the line parallel to this axis generate? What will the line parallel to the axis generate? What figures will the lines perpendicular to the axis generate? What will the rectangle generate?

6. How may a cylinder be generated?

7. How may a cylinder be generated, beginning with a point?

8. If a circle be moved so as to continue parallel to its first position, and so that its center continues in a line perpendicular to the circle at its middle point, in its first position, what kind of a solid will the circle generate?

9. How must a circle be moved to generate a cylinder? On how many of the surfaces of a cylinder can straight lines be drawn?

10. If a right triangle be turned on one of its perpendicular edges as an axis, what will it generate? What will the line perpendicular to the axis generate? What line of the circle will one of the extremities of

the perpendicular generate? The other extremity of the perpendicular will be what point of the circle?

11. If a circle be turned on one of its diameters as an axis, what will it generate? What lines of the circle and sphere are equal?

12. How may a sphere be generated?

13. Can a hemisphere be generated by moving a circle in any manner?

14. How must a semi-circle be moved to generate a sphere?

15. If a semi-circle makes half a revolution on its straight edge as an axis, what will it generate?

1. A cube is a solid, so is any portion of space limited by a surface or surfaces.

2. How many dimensions has a solid?

3. How many dimensions has a part of a solid?

4. How many dimensions has a surface?

5. Is a surface a part of a solid? Why not?

6. Does a surface have thickness? Would several surfaces have thickness? Would several surfaces make a solid? Why not?

WHY ELEMENTARY FORM LESSONS SHOULD PRECEDE THE DIRECT STUDY OF NUMBER.

THE NATURE OF NUMBER—HOW IDEAS OF NUMBER ARE GAINED.— Mathematics is a knowledge of the limitations of quantity. These limitations are of two kinds: form limitations and number limitations. Number is but an abstraction. It is not the quantity nor a quality of the quantity.* That number is an idea which accompanies the sense of sight, touch, or hearing, and is not an impression of the sense itself, was held by Aristotle, Locke, Hamilton and others. The child can not think or compare numbers independent of the quantities which they limit. If we try to think one, or of one, we can not do it, for the oneness does not exist in the adjective, but in that in which the adjective limits. A difference in the force of the adjective good as compared with another adjective good, can not be realized in thought, for the difference exists in the things which these adjectives describe; as, a good boy, a good apple. The difference in the qualities of these adjectives must be sought in an examination of the things limited, and not in the

* "Number in the abstract is, of course, a merely intellectual concept, as Aristotle once and again notices."—SIR WM. HAMILTON.

"It is evident that number, far from being a quality of matter, is only an abstract notion,—the work of the intellect and not of the sense."—ROYER-COLLARD.

adjectives apart from the things. We must proceed in the same way if we would learn to use numeral adjectives, or numbers, intelligently. Until a pupil has a mental picture of an inch cube and of a 3-inch cube he cannot see for himself what part one is of the other; if he cannot see mentally the relation of one edge of the inch cube to one edge of the 3-inch cube, or of 1 edge of the 1-27 to 1 edge of the 27-27, he cannot think the cube root of 1-27. Nor can he compare 1-2 with 1-3 unless he can see their relation to each other through seeing their relation to 6-6, or 1; and to see, or image, 6-6, is to see, or image, the 6 equal parts of some quantity.

THE DEVELOPMENT OF THE IMAGE-FORMING POWER THE BASIS OF MATHEMATICAL INVESTIGATION.—All reasoning in arithmetic is based on seeing conditions, and ability to see conditions is based on ability to think the relations of quantities, and not the relations of numbers, and to see the relations of quantities, the quantities (their correspondents, of course) must be in the mind for examination. The necessary antecedent, then of the formal teaching of mathematics is the training of the imagination;* such a training as will give to the child clearly defined concepts of things which he can recall, analyze, and compare at will. Without clear concepts he has no data upon which to base his reason-

* "If the imagination have not a sound basis in habits of accurate observation, it degenerates into fancy; a term which, though originally it was considered to mean the same thing as imagination, is now used to denote a well-founded difference. Fancy represents the productive or creative power of imagination working without that due restraint of law which is imposed upon its operation by habits of accurate observation, and without that proper and sufficient material of facts which such observation furnishes."—MAUDSLEY.

ing and conclusions. A pupil's real progress in number is measured by his power to think of things independent of their concrete manifestation.. To our losing sight of these facts, and attempting to teach number before a proper basis has been laid by cultivating the image-forming power, may be attributed the dislike to the subject, and the almost incredible inability on the part of a majority of pupils to solve simple problems.

Why Some Ideas of Form, Color, Direction, Position, Size, Etc., Should be Taught Before Beginning the Formal Study of Number.—As clear concepts result from distinct precepts, and these, in turn, from repeated observations, it is evident that the natural approach to number teaching is through things upon which we can fix the attention of the children.

It is true that objects are used in teaching arithmetic in most of our primary schools; but is it true that they are usually so selected, and so used, as to furnish an objective basis for the mind in its operations? Handling objects is of little value if nothing corresponding to them comes into the mind.

Little power is gained to hold an apple in the mind by saying 2 apples and 2 apples are 4 apples. The power to image an apple depends on the ideas you have gained of it, either by direct study or incidentally. It is self-evident that an object of which you know nothing except that there is one or two is not in the mind, and cannot be thought of, and, that the more ideas you have of an object, the more perfect is the concept, and, by the laws of association, the more easily it is held in the mind. If the likenesses and differences of two apples were considered, their color, form, size, weight,

etc., the effect of such a lesson on future mathematical investigation would be far greater than if attention were turned first and solely to the number.

The *number* relations of 4 apples are just the same as the number relations of 4 of anything else. The differences exist in the things. Then to pass from 4 of one thing to 4 of another is to pass to exactly the same thing, if the number only is considered. These repetitions soon become wearisome; the stimulus to thought grows less and less with each repetition, and the interest dies.

Saying that 5 squares and 2 squares are 7 squares does not lead to a close observation of the squares. Children do not see the four equal lines, the opposite lines, the two pairs of parallel lines, the four pairs of adjacent lines, the four pairs of perpendicular lines, the four right angles, the opposite points and opposite angles, and the convergence and the divergence of the lines. These things are overlooked in the consideration of the how many only. As it is with the squares so is it with other objects used for number exercises. Attention must be given to something else in the object if the exercise is to foster habits of investigation which will lead to a living apprehension of the relations of quantity.

I do not here urge the study of the properties of the apple, the square, etc., for the sake of a knowledge of these things, but because I believe it to be the best means of training the observing powers and the imagination, and therefore the most economical basis for the study of arithmetic. Things are held in the mind by their form and not by their number, and a pupil who

studies form and natural science one year, letting number be incidental, and then begins the direct study of number, with these studies in a parallel course, will know much more of number in three years than one who studies number alone from the beginning.

The mere numbering of objects does not build them into the mind; if it did, we could learn form and natural science by merely numbering forms, flowers, etc. Hence, in a typical primary school where balls, shoe-pegs, apples, blocks and toys are used in elementtary number lessons, we find pupils unable to think in number. They can count and repeat tables, but have little power to investigate new conditions — to verify or disprove the conclusions of others. They have not gained objects of thought which they can compare. Saying that 5 beans and 1 bean are 6 beans, that 6 beans less 3 beans are 3 beans, etc., does not awaken an interest in the relations of numbers, but usually degenerates into mere mechanical drill, through which expressions are memorized. *The repetitions necessary to make permanent possessions of the things perceived are not repetitions of words but of ideas, and to get the thought again and again before the pupil is only possible by arousing the mind to activity.*

The Relations of Number Seen through Seeing the Relations of Quantity.—The common practice is to give all the attention to the number and none to the quantity. The converse of this is not recommended, but that the study of quantity should be so pursued as to create a demand for limitation by number. Just as we teach a word when a child has an idea for which it needs expression, so should we teach number when its

help is needed to definitely limit quantities. A child discovers a difference in the length of two lines. There is then a demand for units of measure —units of quantity— and for numeral adjectives that the difference may be clearly defined in thought, and exactly expressed.

Form Better Adapted than any Other Subject for Beginning Systematic Training of Perception and Imagination; Therefore, the Best Basis for Number Study.— Pestalozzi, Froebel, and the kindergartners generally, recognize the value of form both as a direct means of training the perceptive faculties and as a preparation for the study of mathematics.

Geometrical forms are simple in outline and possess distinctive, clearly-marked features which the childish mind can readily grasp. These features are constantly repeated, but in a variety of forms, so that the repititions do not weary. The relations of the parts of the divided solids to the whole and to each other are exact, so that both through analysis and construction facts of form and numbers may be discovered and fixed. Children should begin with the simple and exact, and pass to the complex and indefinite.*

Form furnishes a better means for the beginning lessons in observation than does natural science. Many objects in natural science are irregular —the measurements of their parts not exact. Children can not express definitely the likenesses and differences they discover; hence, their thought to a certain extent is in-

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* "The possibilities of the object must not be too varied, and it must be suggestive through its limitations. The young mind may be as easily crushed by excess as by defect."—Susan E. Blow.

distinct. A description in which only such terms are used as long, slender, short, etc., lacks in precision.† No other study is so well adapted for close comparison in the lower grades as the study of form, because in no other are the likenesses and differences so clearly marked; hence this subject best trains the discriminating power which leads to classification and generalization.

Without elementary ideas of form neither geography, drawing or natural science, can be properly taught. Progress in other subjects will be greatly facilitated by understanding the language of form.

There would be but little difficulty in teaching mensuration, in arithmetic, if pupils were able to think of the forms with which they deal.

Scientific geometry cannot be intelligently taught without elementary ideas of the subject gained through the study of form in the concrete. These ideas should be gained in the perceptive stage of education, and not when the student ought to be exercising his reasoning powers. W. W. S.

"The infant begins to examine forms from the commencement of his existence; for without this knowledge it is doubtful if he could distinguish one object from another, or even be aware of an external world. Gradually he begins to know objects apart and to recognize them, and in time discerns resemblances which cause him to classify them. A vast amount of time and labor is spent by every child in the investigations during the first ten years of his life; but not more than their importance requires. Every child is therefore in some degree a self-taught geometer. Can it then be said that form is not suited for early education?" . . .

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† "It is in mathematical science alone that words are the signs of exact and clearly defined ideas. It is here alone that we can see, as it were, the very thoughts through the transparent words by which they are expressed."—DAVIES.

"Half a dozen simple points investigated and discovered by the pupil will be of more value than a book full of geometry, to which he merely gives a cold assent."

HORACE GRANT.

"Geometrical facts and conceptions are easier to a child than those of arithmetic."

EX-PRESIDENT HILL, of Harvard College.

"Mathematics is the only exact science; if the premises are correct the conclusions must be. To form a strong effectual habit of seeing and thinking of things just as they are and in their exact relations, is the province of mathematics." . . .

"Ideas grow slowly. It takes a long time, with many acts of perception, to fix an idea clearly in the mind. It is of immense importance that these ideas come into the mind so distinctly that they can be used in thinking." COL. F. W. PARKER.

"The elements of geometry are much easier to learn, and are of more value when learned, than advanced arithmetic; and if a boy is to leave school with merely a grammar-school education, he would be better prepared for the active duties of life with a *little* arithmetic and *some* geometry, than with *more* arithmetic and *no* geometry."

PROF. MARKS.

"That pupil is fortunate who has really had good object-lessons in form at and from an early age. . . . In all his teaching he must not forget that the end in view is to produce images of the geometrical figures in the minds of his pupils; so that he and they will be looking mentally at the same or similar objects, and that neither will be lost among the abstract words."

T. H. SAFFORD, Professor of Astronomy in Williams College.

"However excellent arithmetic may be as an instrument for strengthening the intellectual powers, geometry is far more so; for as it is easier to see the relation of surface to surface and of line to line than of one number to another, so it is easier to induce a habit of reasoning by means of geometry than it is by means of arithmetic."

WM. GEORGE SPENCER, the father of Herbert Spencer.

"When the understanding is once stored with these simple ideas, it has the power to repeat, compare, and unite them, even to an

almost infinite variety, and so can make at pleasure new, complex ideas. But it is not in the power of the most exalted wit, or enlarged understanding, by any quickness or variety of thought, to invent or frame one new simple idea in the mind not taken in by the ways aforementioned." LOCKE.

"Instruction must begin with actual inspection, not with verbal descriptions of things. From such inspection it is that certain knowledge comes. What is actually seen remains faster in the memory than description or enumeration a hundred times as often repeated." COMENIUS.

"Observation is the absolute basis of all knowledge. The first object then, in education, must be to lead the child to observe with accuracy; the second, to express, with correctness, the results of his observation." PESTALOZZI.

"If we consider it, says Herbert Spencer, "we shall find that exhaustive observation is an element of all great success."

"The education of the senses neglected, all after-education partakes of a drowsiness, a haziness, an insufficiency, which it is impossible to cure." BACON.

Please read what Agassiz says of the value of comparisons. See page 20.